MECHANICAL ART: GIANT RUBIK’S CUBE
ME 490 RISE
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ABSTRACT
Students, faculty, staff, families, children, alumni, and other visitors to the University of Michigan all make a point to visit the Endover Cube on Central Campus because it is an iconic tradition to spin and enjoy. North Campus does not have an iconic landmark that people come back to see and interact with. Our project team hopes to fill this void with a mechanical art piece that draws inspiration from the Cube, while adding an engineering twist. The design task is to build a giant, fully functional, human-solvable Rubik’s cube art piece to be installed on North Campus. The project is a continuation of the prototyping work begun during Fall semester 2014 as a senior design project for ME 450. The work completed this semester included design iteration, material selection, additional prototyping and testing, and manufacturing three of the 26 final cubelets. We hope to see generations of Michigan engineers collaborating to solve the University’s largest puzzle.
INTRODUCTION
The overarching goal of this project is to design and manufacture a large-scale, fully functional, human solvable Rubik’s cube art piece to be installed on North Campus at the University of Michigan. Thus far, the project has spanned two semesters of design iterations, will continue through the upcoming academic year, and is expected to be completed by May 2016.

Kinetic and Interactive Art
The Rubik’s cube project is not only a mechanical engineering design challenge but also a kinetic and interactive art piece. An example of such art is the Endover Cube on the University of Michigan’s Central Campus, which was in fact an inspiration for this project. Located in Regent’s Plaza next to the Michigan Union, “The Cube” is a 15 foot­tall steel cube balanced on its corner, but despite its large weight, passersby can spin it about its vertical axis with surprising ease. The Cube has become a tradition for Michigan students and alumni to spin and enjoy since 1968. However, North Campus lacks such a symbolic piece, and the Rubik’s cube team took note of this, wanting more for their engineering campus. Before embarking on a long journey towards this project’s completion, the team developed a survey to determine the public desire for a kinetic and interactive art piece such as this for North Campus.

Interest Survey
On September 7, 2014, the Rubik’s cube team released a user interest survey to the North Campus community, in an effort to better understand the public’s opinion on the possibility of a Rubik’s cube art piece joining the large and diverse collection of art on North Campus. The survey asked for interest in the project’s completion, as well as several open-ended questions about what would make the art piece something each user would want to interact with. About 90% of those surveyed expressed interest in the project’s success, and several respondents answered in detail about what aspects of the final product would encourage them to engage with it. These user requirements, such as full functionality, impressive size, and ease of manipulation, have driven the project forward since the survey was released.

ANATOMY OF A RUBIK’S CUBE
There are two classes of relations that are important to consider with regards to a traditional Rubik’s cube toy before approaching a new design for a large-scale art piece. On one level, each piece (or cubelet) of the toy has a physical relationship with the other cubelets, while one the second level, the toy as a whole has a rather complicated relationship with the rest of the world. The first class, or inter­cubelet relationships, are what define the concept of the puzzle and constrain the possible orientations that it can take. The second class, or extra­cube relationships, are what allow the puzzle to be manipulated by its user(s) in an intuitive way. Through careful consideration of each class, we were able to begin the design of our large­scale Rubik’s cube.
**Inter-cubelet Relationships**

As seen in Figure 1, a Rubik’s cube is comprised of three different types of cubelets. At the center of each of the six faces there is a center cubelet. The center cubelets act as the keystones of the Rubik’s cube, as they hold all of the other cubelets from falling out through the use of the cylindrical surfaces on their interior. Adjacent to each pair of center cubelets is an edge cubelet. These cubelets each have two small protrusions, or tabs, that are captured by the aforementioned cylindrical surfaces on the center cubelets. They also each have their own cylindrical surfaces located on either side of the two tabs. These surfaces are used to hold in the third type of cubelet, the corner cubelet. There are eight corner cubelets, one located between each trio of edge cubelets, and they have three tabs on the inward-facing side that are constrained by the three edge cubelets they each touch at any given time.

![Figure 1: Exploded view of Rubik’s cube toy, showing three types of cubelets, and three pairs of axes around which the six faces rotate. [1]](image1)

![Figure 2: Six center cubelets fastened to the core, each allowed to rotate independently. [2]](image2)

In order for the six center cubelets to hold in the rest of the pieces, they must be attached to one another by some means that still allows each center to rotate independently of the others. Figure 2 shows the centers with all edge and corner cubelets removed, revealing the core to which all of the centers are bolted. Made evident by this view is the fact that opposite center cubelets always remain opposite one another; while the edge and corner cubelets can travel around the rest of the cube, the color of each face is determined by the center cubelet. One important consequence of this relationship will come into play when designing the mounting structure.
**Extra-cube Relationships**

A standard Rubik’s cube toy is both small and light, meaning that it can be held in the hands and oriented easily with no mount or stand needed. However, a cube of “impressive size,” as required by our interest survey, would be far too heavy for any mortal being to hold. For this reason, the support structure was one of the first parts of the concept to be considered. As the Endover cube was an early benchmark for the art piece, mounting the Rubik’s cube by a corner would mirror its aesthetic well. However, the corner cubelets are not attached to the rest of the cube, so we decided that mounting the art piece to or through the centers would be a priority.

![Figure 3: Mounted Rubik’s cube concept, showing that slices A, B, and C can rotate independently from each other about main shaft Z.](image)

Figure 3 shows the chosen mounting concept, which mimics a globe. A main shaft Z runs through two opposite center cubelets, which are able to rotate independently on the shaft. Thus all of the degrees of freedom necessary to the puzzle are maintained, while the heaviest and most structural part of the cube is supported directly by the mounting shaft. Detailed analysis of this concept is discussed in the main shaft deflection section.

**INITIAL DESIGN ITERATION**

**User Requirements**

As a result of preliminary research and feedback from the user interest survey, a set of five user requirements was created. We determined that our design should be safe and stable; fully functional; robust, durable, and maintainable; impressively large; and easy to manipulate, ergonomic. These requirements drove the decision making for the first version of our design.

*Safe and stable:* Clearance gaps between the corners of the cube and its mounting stand were designed so that no pinch points were created as the cube spun about the main shaft. Additionally, the initial design version was designed to have no gaps between adjacent cubelets.
This ensured that fingers and other appendages were safe from getting stuck in small cracks, although it had the downside of increasing friction wherever two interior faces touch.

**Fully functional:** The center cubelets were designed to rotate freely about their respective shafts, while being constrained in the axial direction. To maintain radial alignment of each face, a series of rollers were designed to replace all of the tabs found on a Rubik’s cube toy. This design change greatly reduced frictional forces present when a face is turned due to the decrease in coefficient of friction achieved by switching from sliding to rolling friction.

**Robust, durable, and maintainable:** In order for this art piece to last for generations on Michigan campus, we realized that it must be able to withstand the rigors of daily use without significant deterioration. This was taken into careful consideration when choosing the materials for the cube. However given that parts do eventually wear out, we also designed most parts to be interchangeable so that they could be replaced with modest cost and effort if necessary.

**Impressively large:** This design criteria was drawn directly from the user interest survey, and required some quantification. We researched national human wingspan dimensions, and decided to design the cube with an edge length equal to the 10th percentile wingspan of adult US women. Thus 90% of female and over 99% of male users would be able to reach across one face, ensuring that the design is as large as possible without seeming excessively daunting to approach as a single user.

**Easy to manipulate, ergonomic:** Another criteria taken from the user interest survey, ease of manipulation required some empirical testing to quantify. Team members pulled on a variety of different objects and rated the difficulty on a 1-5 Likert scale. These ratings were compared with the forces exerted during each pull, and the target force to turn a side was set at $18 \pm 4$ lbs. The design was also made without any sharp edges to ensure that it would be comfortable and safe to use.

**Initial Material Choices**
The five user requirements played a large role in the choice of materials for the initial design version. The cube was designed with a lightweight welded aluminum frame which would eliminate the need for most fasteners. Curved surfaces on the inner faces of the center and edge cubelets were designed to be rolled aluminum sheet, which would be welded onto the rest of the frame. The rollers were Ultra high molecular weight polyethylene (UHMWPE) to take advantage of its high impact resistance and extremely low coefficient of friction. Finally, the colored exterior of the cube was designed as thermoplastic paneling that would be heated and formed around the aluminum frame after welding was completed. This would create a seamless, professional appearance to the outside of the cube.
**Design Issues in Initial Design**
Several critical issues were identified through the prototyping and analysis of the first version design, particularly in its ease of manipulation. A prototype edge cubelet was fabricated using the initial design, and while the main points of functionality were achieved, the tolerances were not satisfactory. Misalignment of cubelets in an assembled cube thus became a serious concern, as even a small amount of misalignment can stop a face of the cube from turning. A rough engineering analysis was then performed to estimate the force that would be required to turn one face, and it was more than double the target of 18 ± 4 lbs. These problems stemmed from an unrealistic manufacturing plan, which was addressed thoroughly in the second version design created through ME 490. We discovered through building the prototype cubelet that our level of welding expertise was far too low to allow welding as a serious possibility, and the elimination of fasteners turned out not to be particularly helpful because local failures would have to be fixed by replacing an entire cubelet rather than only replacing the failed component.

**RISE DESIGN CHANGES**

**Motivations**
ME450 provided an opportunity to get the project off the ground and identify a preliminary design. One of the primary goals for ME 490 RISE was to completely update the design as the preliminary design failed to meet the most important user requirements and had several manufacturing problems. In order to create a more viable design, the several issues needed to be mitigated including reducing the force required to turn a face, tightening tolerances, improved manufacturability, modularizing the design, better panel attachment, and improved cubelet alignment.

**Design Update: Structure**
Two main design changes were made to adjust the structural design of the cube. The first and arguably most comprehensive design change was to switch from welded aluminum tube stock to t-slotted extruded aluminum from 80/20 inc. This allowed for the elimination of welds in favor of a simple bracketing scheme. This design change allowed for significantly more flexibility in structural and component design. The eliminated welding also reduced concerns about the general lifetime of the art piece due to weld fatigue. Additionally, the team has very little welding experience, amplifying the concern of creating a robust structure. By using a simple bracketing scheme, the manufacturability has greatly increased and provided a huge boost to assembly efficiency and speed. Extruded aluminum has small slots that allow for simple and easy panel attachment, and the substructures are can be removed and adjusted providing better modularity.
The second structural change involved updating the design of the complex curved internal surfaces in the cube. In the initial design, sheet metal was rolled and welded to aluminum splines to create curved surfaces. This method of attachment created poor tolerances and nearly impossible repeatability. The new design eliminated the sheet metal and splines in favor of CNC milled solid blocks of aluminum.

![Figure 4: Large format CNC mill machines curved surfaces into solid block of aluminum.](image)

Although the cost and manufacturing time increased greatly with this design change, the curved surfaces are a vital part to the mechanical functionality of the art piece and warrant the precision provided by the mill. These surfaces act as a guide for the rollers and are packed closely together while moving past each other in alternating planes. Small differences in the surface dimensions could create a critical failure in the rubik’s cube mechanical functionality and updating the design eliminated this. Using a CNC setup, very tight tolerances and near perfect repeatability could be achieved. Additionally, using the CNC mill allowed the team to create the necessary complex geometries in the design and allowed for simplifications and improvements in attachment schemes.

**Design Update: Concept**

The second area of design changes were made to the higher level mechanical concept for the Rubik’s Cube. A set of internal surfaces were added to the corner and edge cubelets in order to fully constrain the cubelets. This new functionality was perhaps the most fundamentally important design change as it resolved a number of difficult problems including greatly reducing the force required to turn a face and improving cubelet alignment. The new internal surfaces concentrate the frictional forces to the inside of the cube where they are mitigated through the use of rolling contact. The surfaces fully constrain both edge and corner cubelets inside the cube structure and remove pressure between cubelets. This creates gaps between the cubelets and greatly helps with cubelet alignment (Figure 5).
Figure 5: The edge cubelet in the previous design (left) was held in place by its rollers and edge surfaces, while the current design (right) includes a new structure on the inside of the cubelets that rests on internal surfaces, constraining the cubelet inside the cube.

As seen in the original design, the edge cubelet is constrained from falling outwards by the rollers, and is constrained from falling inwards by the edge surfaces—these are the surfaces that the panels are attached to, creating sliding contact and high frictional forces. In the new design, the edge cubelet has a new structure created in the inside of the cubelet that rests on internal surfaces. This constrains the cubelet inside the cube and relieves the outer surfaces where the panels attach through gaps.

Center Cubelet: The center cubelet (Figure 6) is at the top of the cubelet hierarchy. Six center cubelets in each of the axial directions directly restrain the edge cubelets and indirectly restrain the corner cubelets. The yellow surface in the Figure showcase the CNC milled aluminum blocks. A series of belleville washers and a bearing nut allow the center cubelets to be adjusted and translated axially in order to adjust the tension of the entire cube.

Figure 6: Center cubelet design

Edge Cubelet: The edge cubelet (Figure 7) is second in the cubelet hierarchy. Twelve edge cubelets directly restrain the corner cubelets. The edge cubelet is designed with CNC milled
aluminum surfaces that mesh directly with the center cubelet surfaces. Each edge cubelet has two sets of two rollers that run along the curved surfaces of center and edge cubelets in a different plane. Two sets of four ball transfer bearings run along aluminum plating and greatly reduce frictional forces inside the cube.

Figure 7: Edge cubelet design

**Corner Cubelet**: The corner cubelet (Figure 8) is third and last in the cubelet hierarchy. Eight corner cubelets are restrained by edge cubelets but do not in turn restrain any pieces themselves. Similar to the edge cubelets, the corners cubelets have three sets of two rollers and three sets of three ball transfer bearings. The rollers and bearings serve the same purpose as described above in the edge cubelet.

Figure 8: Corner cubelet design
**Interactions:** As seen in Figure 9, the curved surfaces contributed by both the edge and center cubelets work together to create a single circular surface (yellow surfaces seen in part A). This surface serves as the guide for the rollers (seen in part B) from the edge and corner cubelets to travel along. Additionally, flat surfaces (grey surfaces in part A) contributed by the edge and center pieces create a plane like surface at the base of the yellow circular surface where ball transfer bearings (seen in part B) contributed by edge and corner cubelets run along.

![Figure 9](image)

**Figure 9:** Curved surfaces contributed by edge and center cubelets work together to create a single circular surface, guiding the edge and corner cubelets’ rollers. Similarly, the flat surfaces contributed by the edge and center cubelets provide a flat plane for the ball transfer bearings to travel on.

The complex interactions of all the cubelets in motion is best understood through an animation. A video demonstrating how the design works can be found at this link: [https://www.youtube.com/watch?v=C2axW4u6Tqk](https://www.youtube.com/watch?v=C2axW4u6Tqk).

**ENGINEERING ANALYSIS OF DESIGN**

Engineering analysis of both the previous and the current design was completed to ensure that the cube is structurally sound. This analysis included determining the effect of deflection of three main components: the center cubelet support shafts, the main shaft, and the roller support shafts of the previous design. This last analysis contributed to iterations of the roller design. The final analysis completed was determining the force required to turn a face of the cube.
**Gap Closure Between Cubelets**

The center cubelet support shaft, identified in Figure 10, supports significant weight from the center cubelet and from the eight surrounding cubelets of that face. The shaft is rigidly fixed to the core, however the free end of the shaft will deflect under loading. The ¼” gap between adjacent cubelets will be reduced by the deflection of the center cubelets. This deflection has two contributing factors, which are vertical and rotational displacement, as shown in Figure 11.

![Figure 10: A cross-sectional view of one face of the cube. The red identifies the center cubelet and its support shaft. The arrow indicates the ¼” gap between two cubelets.](image)

 Beam theory was used to determine the total deflection of the center cubelets and therefore gap closure. It was assumed that the center cubelet support shafts would need to support 500 pounds, a conservative estimate of the weight of one face of the entire cube. It was also assumed that this loading would be on the free end of the shaft, which is also a conservative assumption because in
the actual design the load is partially distributed along the shaft. Equation 1 was used to
determine that the displacement due to vertical deflection is 0.0965 inches.

\[ \frac{d^2v}{dx^2} = \frac{M}{EI} \quad \text{Eq. 1} \]

The displacement due to rotation was found using the theoretical slope of the end of the shaft. This displacement was calculated to be 0.0435 inches resulting in a total displacement of approximately 0.14 inches. Based on this analysis, the deflection of the center cubelet support shaft will cause a conservative gap closure between cubelets of 56%, which will not compromise the functionality of the cube.

**Main Shaft Deflection**

A second concern regarding deflection of structural components is the deflection of the main shaft. The main shaft is a 2 inch diameter steel rod that runs through the entire cube, as shown in Figure 12. It must be strong enough to support the weight of the cube and must not deflect such that the cube’s functionality is compromised. It is also a design requirement that the main shaft does not deflect such that the corner cubelets contact the stand, which would prevent the cube from rotating about the main shaft.

![Figure 12](image)

**Figure 12:** The cube mounted on its stand. The main shaft is identified in red.

![Figure 13](image)

**Figure 13:** A free body diagram of the main shaft.

The free body diagram of the main shaft, shown in Figure 13, was used along with Equation 1 to calculate that the maximum deflection of the main shaft is 0.57 inches. This is a very conservative estimate and will be reanalyzed during the next design phase, which will include addressing the stand design.
Finite Element Analysis of Roller Shafts

The previous edge cubelet design included rollers cantilevered off the end of welded aluminum shafts (Figure 14). These rollers would take significant loading from adjacent cubelets. The design raised concerns about roller deflection and weld failure.

A finite element model was created to determine the effects of loading on the rollers (Figure 15). The FEA model concluded that the design would survive the loading with a safety factor of 6. Despite the theoretical success of the previous roller design, the concern about the roller shaft deflection and weld failure was mitigated with a design change. In the revised design the rollers are no longer cantilevered off of welded aluminum and are now doubly supported on both ends, as highlighted in Figure 16.
Force Required to Turn a Face

The final analysis conducted was the force required to turn a face of the cube. This is crucial to design success because it directly affects both the user experience and the functionality of the cube. As a benchmark, the team determined that the force required to spin the Endover Cube on Central Campus is $30 \pm 3$ lbs. Based on this and other experimental testing, the target force required to turn a face of the Giant Rubik’s Cube was set at $18 \pm 4$ lbs.

Based on the previous design, a conservative analysis model was constructed (Figure 17). This model assumes that the sliding contact between two layers of eight cubelets is represented by the conservative disk area identified in Figure 17, rather than the smaller actual contact surface area.

![Figure 17: A diagram of the surface area used in the calculation of the force to turn a face of the cube in the previous design.](image)

It was assumed that the friction forces dominate the dynamics of the face rotation, which results in Equation 2 as an approximation of the force required to turn a face, $F_{\text{turn}}$. The friction force is affected by the weight of a face of the cube as well as the tensioning force introduced by the Belleville washers and nuts on the center cubelets. In this analysis, an assumption was made that the tensioning force equals the weight of a face. Therefore, $F_{\text{face}}$ is the effective weight of a face, which includes the tensioning force (Equation 3).

$$\tau = \frac{2}{3} F_{\text{face}} \mu R = F_{\text{turn}} R \quad \text{Eq. 2}$$

$$F_{\text{face}} = Mg + F_{\text{tension}} \quad \text{Eq. 3}$$

The team experimentally determined the coefficient of friction to be 0.23. Based on Equation 2, this results in a required force of 37.5 lbs. This is greater than both the target force of 18 lbs and the 30 lbs required to spin the Endover Cube. The team decided to replace the sliding contact in the previous design with rolling contact in the current design to reduce the concerns raised by the above analysis. This design iteration was implemented by the addition of the ball transfer bearings.
TESTING

Purpose
Although all sliding surface contact in the previous design was converted to rolling surface contact in the current design, a new potential conflict arose due to the combination of ball transfers and necessary small gaps between cubelets. If one or more of these ball transfers were to be caught between the gaps between the cubelets’ inner surfaces, the question of how much force is required to push past these gaps becomes critical to the design’s success. For this reason, a test rig was designed and built to explore the relationship between the size of the ball transfers, the size of the gaps, and how much force is required to manipulate a virtual face of the cube.

Construction and Setup
As shown below (Figure 18), a cube face of nine cubelets consists of a plane of ball transfers (marked in blue) on one side and a plane of machined aluminum with ⅛” gaps (marked in red) between the cubelets’ inner surfaces.

Figure 18: The rig to test the force required to free the ball transfers from small gaps mirrors the layout of ball transfers and small gaps located on an internal face of the cube. Red lines mark the gaps between cubelet inner surfaces, while blue dots mark ball transfers.

This configuration was mirrored in the test with a plane of ball transfers bolted to a sheet of plywood and laser cut acrylic panels bolted to another sheet of plywood. The acrylic panels were designed with slots such that there was freedom to examine multiple gap widths in the test setup.
These two parts were slid onto PVC tube stock to keep both faces concentric. To represent the force between the two actual faces in the cube created by tensioning from the Belleville washers and cubelet weight, human volunteers stood on top of the test setup.

**Results**

Because this test uses some materials that differ from actual materials in the cube design, this test serves as a qualitative rather than quantitative measurement of the force required to free the ball transfers from the gaps. Even with about 250 pounds of force between the faces, the force required to rotate one plane along the other was small enough to be considered qualitatively “easy to turn,” as was a user requirement called for by the user interest survey. This subjective measurement was repeated with users outside the Rubik’s cube team to prove the validity of the results. Additionally, users noticed that with each 90 degree turn of the test setup, they felt a satisfying click as the ball transfers found the gaps and the faces aligned. This feature of the test rig was not considered in designing the actual cube, but because of the positive feedback, the next design iteration is expected to include a similar characteristic. The plan is to add indentations to the currently flat aluminum surfaces that will capture the ball transfers when the cube’s face has been turned a full 90 degrees and is aligned well with the rest of the cube. Not only will it improve the user experience by adding to the tactile experience, but it is expected to promote alignment of the cube’s faces, making solving the cube physically easier.

**EDUCATIONAL OUTCOMES**

This project has many dimensions beyond that of a traditional engineering education. The Rubik’s cube team had to take these considerations into account in order to be successful in short and long terms. The facets of this project have included defining a complex problem, developing and exercising engineering skills, applying project management concepts, and communicating effectively.

**Defining a Complex Problem**

At the outset of the project, the Rubik’s cube team knew the project was a large undertaking. As is intrinsic to a complex design problem, setting goals is no simple task. Realistic goal-setting for this project was an ongoing and challenging consideration. For instance, at the outset of the semester, the team was under the impression—or delusion—that the entire cube would be built by the Design Expo and Mechanical Engineering Undergraduate Symposium on April 16th of this year. Instead, the team took more time than expected to complete design changes essential to the cube’s success, and a new goal replaced the original. The team would present one of each of the three types of cubelets, fully manufactured, at the Expo. Measuring success or failure in a project of this scale, complexity, and depth is another sophisticated and intricate question, one that troubled the team throughout the semester, but at this point the team feels a few steps closer to understanding.
**Engineering Skills**
The engineering skills learned and built upon during the journey of this project are countless. As previously discussed, the team applied analytical methods learned in undergraduate courses such as strength of materials (ME 311). The team largely built upon experience from previous work in the design and manufacturing track in the mechanical engineering department, including simple machining processes, computer aided design, and finite element analysis. During this term, the team also had the opportunity to learn new engineering skills, the most prominent of those being the use of computer numerical control (CNC) milling machines. Finally, this course impressed upon the team what other design courses have been unable to thus far—the importance of design iteration. Because time was taken this semester to reconsider aspects of the previous design concept and explore ways in which to solve its problems, and all this was done at such depth, the team realized how important design iteration is in real-world engineering.

**Project Management**
Skills required to manage a project were key this semester. With many moving parts needing to come together simultaneously, the team decided it would be appropriate to work in parallel much of the semester. This required efficient timelining and organizational skills. Of course, ordering parts, often custom-made, from external vendors lent itself to delays and other unforeseen obstacles. Surmounting these difficulties taught the team that clear and frequent communication would be an invaluable tool, even after this project’s completion.

**Communication**
Throughout the term, the team kept in contact with many types of people important to the Giant Rubik’s Cube’s realization. The team communicated with potential sponsors, motivating them to support not only our goals but what the final product can do for North Campus. Similar to this was the team’s networking with key stakeholders in the College of Engineering. Gaining the support of Dean Munson was crucial to the eventual installment of the art piece. During the redesign phase, the team had recurrent conversations with experts in fields varying from panel material selection to specialized bearing applications. Additionally, keeping in continual contact with vendors and suppliers has helped to keep the team’s timeline as accurate as possible. Finally, the team’s work with machinists has hinged upon clear communication between them. From drawings to wordy explanations, the team has depended upon their ever-growing communication skills to machine their own custom parts.
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